

# A nearly linear asymmetric GARCH model

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A new class of nearly linear generalized autoregressive conditional heteroskedasticity (GARCH) models is proposed. The idea is to employ the softplus function used in deep learning as response function to the conditional variance in GARCH models. The proposed class of models spans linear and nonlinear GARCH models. The softplus GARCH (spGARCH) model allows for negative parameters while ensuring the conditional variance is nonnegative. The softplus function smoothly approximates the rectified linear unit function, maintaining differentiability of the conditional variance. Using the theory of general nonlinear GARCH processes, we prove the existence of a unique strictly stationary solution of the spGARCH process which is ergodic. We show that the quasi maximum likelihood estimator (QMLE) of the parameters of the spGARCH model is consistent and asymptotically normally distributed under regularity conditions. The spGARCH model is extended to allow for an asymmetric effect of a positive and negative lagged value of the process on volatility. We present an application to German stock returns.